The influence of surface roughness on fluid flow through cracks

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Received in final form February 2002

ABSTRACT Leak-before-break (l-b-b) safety cases depend on predictions of flow rate through postulated cracks. The calculated flow rates are dependent upon assumptions made about a number of features including fluid friction, and this in turn is influenced by surface roughness and flow regime. This paper considers the uncertainties associated with flow rate prediction in both the laminar and fully rough turbulent regimes as influenced by fluid friction. It shows how uncertainties can be bounded. In particular it discusses the maximum values for fluid friction that might arise in practice. The use of computational fluid dynamics in future analyses could significantly reduce the uncertainties associated with fluid friction in cracks.

Keywords fluid friction; flowrate calculations; leak before break; laminar flow; turbulent flow.

INTRODUCTION

Safety cases for pressurized equipment can include arguments based on the leak-before-break (l-b-b) concept. The basic premise is that any developing defect would grow through the wall of the pressure vessel, and the leakage be detected, in advance of any potential catastrophic failure. It is not an objective of this paper to discuss the numerous caveats and requirements of such assessments, but simply to concentrate on facets of leakage through postulated defects in pressurized equipment. In particular the paper will concentrate on aspects influencing fluid friction.

When considering l-b-b safety cases there are conflicting interests with regard to the calculation of leakage rates as now discussed.

In order to ensure that a conservative approach is taken, it is necessary to assess lower bounds to flow rate in order to ensure that the postulated leakage can in fact be detected. On the other hand undue pessimism may mean that l-b-b arguments cannot be sustained because of detection difficulties. There is also an interaction between detection systems and predicted flow rates. If predictions are inaccurate the selected detection system may be suboptimal, or, at worst, not effective.

At the other extreme, associated consequences arguments (discharge into restricted areas, consequences of loss of pressure, etc.) require upper bounds to flow rate to be calculated. Again, undue pessimism is not required, as effort may be invested to mitigate against an event that will not arise.

Overall, the best solution is to have the best estimate of flow rate. This output will meet all requirements. It is, however, an option that is unlikely to be achievable, and the optimum will be to minimize the range between the upper and lower bounds.

This paper will discuss various aspects that influence the calculation of flow rate through cracks, and will concentrate on fluid friction. This is the interaction between the fluid and the crack wall and the paper will consider how this is treated in various models.

FLOW RATE THROUGH CRACKS

Flow rate through cracks is dependent upon a large number of factors. In simple terms:

$$Q_m = f(G, F_p, \Delta P, \lambda) \tag{1}$$

Here, Q_m is the mass flow rate that is a function of G, the geometry, F_p, the fluid properties, the pressure drop, ΔP, and a dependence upon λ, the interaction between the fluid and the surface of the flow path.

In practice, it is not possible to accurately define the geometry. Using fracture mechanics, it is possible to

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predict the macroscopic geometry of the postulated crack and this will define crack width, length and depth. It is normal to calculate a critical crack length and then to ensure that an adequate margin exists to avoid catastrophic failure. Uncertainties will exist over actual material properties, and in particular fracture toughness and its variation with crack extension. Further, cracks do not always grow in a plane, but can deviate. Width will be a function of length and may vary through the thickness. Depth is usually assumed to be the through wall thickness of the vessel. However, there is the prospect of deviation.

The fluid properties, F_p, are not always accurately known. If the fluid is a liquid, and remains so throughout the flow process, then the fluid properties can be reasonably well defined, as also can the flow process. If a transition to a gaseous phase can result, i.e. water to steam, then both the flow process and the fluid properties are more difficult to define. For gaseous flows, the properties will depend upon the thermodynamic process. Isothermal and adiabatic processes can usually bound the extremes, although neither process is sustainable in practice. For relatively narrow cracks, measurements show that there is little temperature drop along the flow path. Heat is fed into the expanding gas through the wall of the vessel and via friction losses. Hence isothermal assumptions are not unreasonable, and such calculations have the merit of being relatively simple. For larger cracks there will be some temperature drop along the flow path. However, comparative assessments have shown that the differences between flow rates calculated using isothermal and adiabatic assumptions are not large. Nevertheless a realistic fluid dynamics model is required for the process and fluid being considered.

While the overall pressure drop will be known, the actual pressure drop across the flow path may be less than this for gaseous and transitional flows where choking can occur. However, the fluid mechanics models will address this issue.

It is seen from the above that an accurate prediction of flow rate through a postulated crack is not feasible. There will be a great deal of uncertainty associated with any prediction although bounding calculations are possible. The above has not addressed the question of the interface between the fluid and the wall of the crack (or fluid friction) and this will be addressed next. However, the preceding discussion shows that large uncertainty will exist in any l-b-b flow rate calculation, irrespective of uncertainty in fluid friction.

FLUID FRICTION

Historical approach

A lot of effort has been expended in trying to understand the interaction between a fluid and the surface over which it is passing. Historically the work has been empirical and was driven by the requirements of aerodynamics and hydrodynamics. The situation is changing as computational fluid dynamics (CFD) becomes more accessible, and this will be referred to later in this paper. Much of the classical work was initiated in the 1930s and an accessible summary appears in Ref. [1]. Included in this category of work is that of Nikuradse who conducted a series of flow experiments along pipes coated with uniform sized sand grains. From his fluid flow analysis he was able to determine fluid friction coefficients, λ[a], which correlated with Reynolds Number (*Re*) when reduced to the simple ratio of pipe radius to sand grain size, Fig. 1.

The left-hand side of Fig. 1 shows λ to be independent of roughness; this is laminar flow. On the right-hand side of the diagram is the fully turbulent regime where λ becomes independent of *Re*, but varies with roughness.

Laminar flow

It is not the intention of this paper to discuss laminar flow at any length. However, there are certain features that are pertinent to subsequent discussions.

The work reported by Schlichting[1] shows that for laminar flow all data reduced to a single line relationship between friction factor and *Re*. However, the highest value for λ shown in Fig. 1 is 0.12. Jones *et al.*[2] looks at laminar flow through rough natural fractures as occur in rock. Here a dependence of friction factor on roughness is observed and with λ-values of some 100 recorded. Button *et al.*[3] also reports a similar dependency on roughness.

Turbulent flow

In the fully rough regime depicted in Fig. 1, the following correlation applies.

$$\lambda_N = \left[2.25 \log \frac{R}{K_s} + 1.74\right]^{-2} \tag{2}$$

Here λ_N is the friction factor as derived by Nikuradse, R is the pipe radius and K_s the sand Roughness. Between the laminar and fully rough regimes is the transition region. It is important to note that there is a lower bound to the data and this, attributable to Blasius (Ref. [1]), is given by:

$$\lambda_B = 0.316 Re^{-0.25} \tag{3}$$

Here λ_B is the friction factor as derived by Blasius and *Re* is the Reynolds Number. Another important finding was that data such as those in Fig. 1 could be used for

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[a] Note that in some literature f is used. The normal relationship is $\lambda = 4f$.

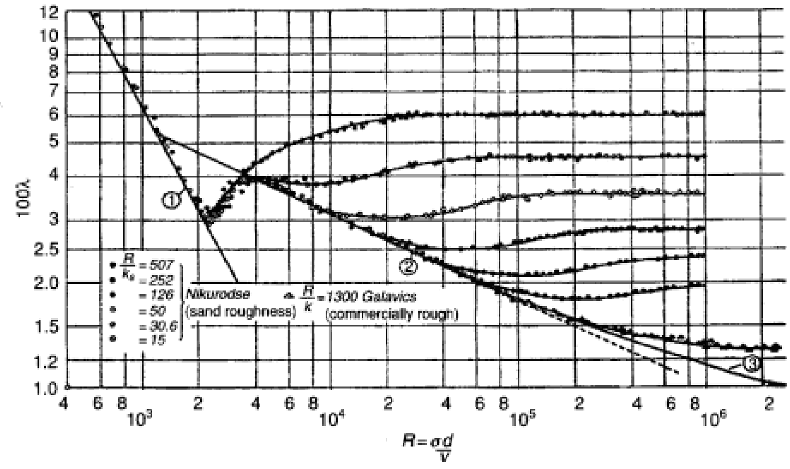

Fig. 1 Relationships between fluid friction and Reynolds number as a function of sand roughness (from Ref. [1]).

noncircular geometries using the hydraulic diameter, d_h, concept, where

$$d_h = \frac{4A}{P} \tag{4}$$

A is the cross sectional area of the section and P is the wetted perimeter.

For a crack of width w and length l and with $l >> w$ then

$$d_h \approx 2w \tag{5}$$

Fluid friction in cracks

In cracks the roughness can take up a large percentage of the surface separation. Also, as mentioned earlier, the flow path is not necessarily straight, as crack growth can deviate from a simple plane. The nature of the roughness will also depend upon the nature of the cracking process. If growth is intergranual then notionally conformal surfaces result. For a fatigue process the surfaces are more random.

It is possible to consider two components contributing to the effective friction in flow through cracks. One is tortuosity that arises as a consequence of cracks deviating from a simple plane. This results in losses as the flow changes direction. The second component arises from the interaction between the roughness itself and the fluid.

In general, the treatment of friction in l-b-b calculations is simplified in that only two regimes are considered. If *Re* is less than some 2000 then flow is assumed to be laminar. Above that it is assumed to be fully rough. For most situations this is conservative in terms of obtaining a lower bound to flow.

The computer program SQUIRT[4] estimates flow rates through cracks and its thermo-hydraulic model covers two phase flows, specifically water to steam. This program treats fluid friction in two parts, tortuosity and roughness. The roughness component is based on Eq. (1). The problem is knowing how to input the tortuosity, and some advice is available based on empirical data. However, it may be argued that the macroscopic tortuosity can be absorbed with the microscopic tortuosity that exists. The major effect being to extend the flow path length.

In the DAFTCAT program[5] only roughness is considered, but freedom exists to extend the effective flow path length if required.

The correlation between surface roughness and fluid friction used in DAFTCAT (λ_D) is based on a series of experiments (Ref. [3]). These used nominally rectangular crack shapes with shot blast surfaces. Nitrogen was used as the fluid. In these experiments, the surface was characterized in terms of the R_a value. In the fully rough regime it was found that:

$$\lambda_D = \left[2.25 \log \frac{w}{R_a} - 0.573\right]^{-2} \tag{6}$$

Equation (6) is compatible with Eq. (2) if

$$6 \leq \frac{K_s}{R_a} \leq 12 \tag{7}$$

These relationships are illustrated in Fig. 2. If K_s is considered to be the diameter of spherical sand particles, then the close packed surface is an array of hemispheres and the theoretical relationship with surface roughness is

$$\frac{K_s}{R_a} = 8 \tag{8}$$

The agreement between Eqs (7) and (8) is reasonable and gives confidence in the correlation developed.

A large range of flow rates and crack widths are covered in the work reported in Ref. [3] and included both

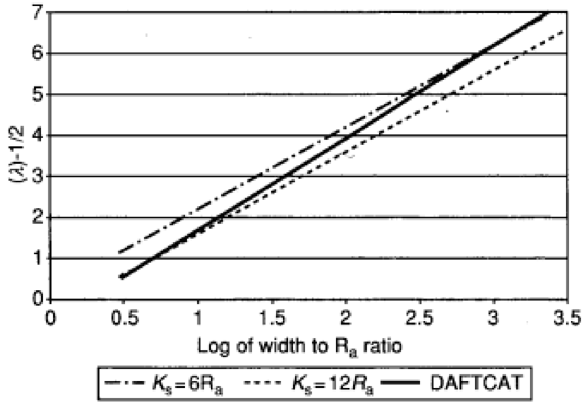

Fig. 2 Comparison of DAFTCAT and Nikuradse friction coefficients for different K_s to R_a assumptions.

laminar and turbulent flow. However, unlike the work of Nikuradse, the laminar flow data did not reduce to a single line and a dependency on absolute separation was found. This finding was in general accord with that reported in Ref. [2] and will be returned to subsequently.

Gardiner and Tyrrell[6] reports the results of experiments on macroscopic representations of cracks. Conforming artificial rough surfaces were created and the separation between the two mating surfaces was scaled to give an assembly representative of what might be expected in a cracked pressure vessel. Two geometries were used.

The first geometry was based on a block comprising granite chips set in Plaster of Paris. Cleaving the block created a fracture surface. The created surface was then used to produce matching crack faces for the experiments. It can be argued that this surface would be representative of crack formation from an intergranular process.

The second macroscopic geometry was more complex and less representative of a real surface. The authors describe it as the Giants Causeway, because its geometry is similar to that of the geological formation. It is therefore a very angular surface with a large number of discontinuities. One surface was produced and a matching conforming surface produced to form the crack.

For both surfaces, experiments were conducted over a range of separations. In both sets of experiments it was seen that the laminar flow data did not reduce to a single curve. In the fully rough regime, only the 'Giants Causeway' geometry yielded sufficient data to determine a formulation similar to that in Eq. (2), viz.:

$$\lambda_G = \left[1.935 \log \frac{w}{R_a} - 0.705\right]^{-2} \tag{9}$$

This formulation assumes that R_a for the surface is 4.5 times the roughness height, b (5.77 mm).

Bounds to fluid friction

Turbulent flow

Equations (2), (6), and (9) are empirical and hence have no bounds defined. Clearly λ must be positive. However, use of Eq. (3), i.e. assuming smooth surface, will give a more meaningful lower bound.

Upper bounds are more difficult to quantify. Neither the work of Nikuradse, nor that in Ref. [3] indicated that maxima were being approached. However, the work reported in Ref. [6] using nesting macroscopic roughness (i.e. the surfaces could be closed completely together) showed that as crack separation was reduced and the roughness began to overlap so λ approached a maximum and then began to reduce as the surfaces were moved closer together.

Two very different artificial surfaces were used in the experiments and yielded maximum values for λ of approximately 1 and 4 for the intergranular type surface and the Giants Causeway, respectively. The existence of a maximum dependant upon the geometry and *Re* was addressed and a theory for the limiting behavior advanced based on flow through packed beds. [It is noted that Ref. [2] (although concentrating on laminar flow) shows flow through porous media to be an upper bound to friction factor.] The correlation between the predicted limits and the experimental results is good for the Giants Causeway crack, but poor for the other.

Nevertheless, the evidence is that maxima do arise in the value for friction factor, and that the peak is associated with overlap of the surface roughness.

Equation (6) is based on a wide range of experiments, and uses the ratio of mean separation of the surfaces to the surface roughness as represented by R_a. The relationship between peak to valley heights on the surface to R_a depends on a lot of factors but is probably bounded[7] by:

$$4 \leq \frac{b}{R_a} \leq 9 \tag{10}$$

Overlap between the opposing surfaces of a crack will commence when the separation, w, is equal to the roughness height. Substituting the relationship of Eq. (10) in Eq. (6) and setting w equal to b results in the relationship shown in Fig. 3. However, Fig. 3 can only be used if the relationship between b and R_a is known. The lower value of 4 used corresponds to a triangular array and is very unlikely to be representative of a real crack surface. Real cracks will have b/R_a ratios greater than 4. It is seen that λ reaches 1.6 with $b/R_a = 4$. The actual peak in fluid friction is expected to be marginally greater than that associated with the values shown in Fig. 3. However, comparison with maxima actually measured suggests that the curve is a reasonable bound. For example for

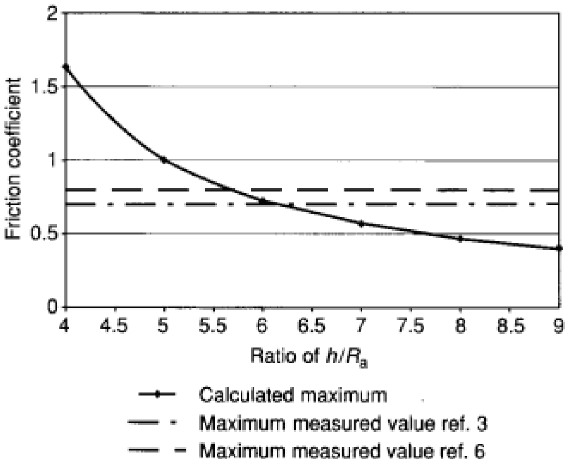

Fig. 3 Comparison of calculated maximum fluid friction coefficient and experimental results.

the more reasonable geometry used in Ref. [6] the maximum value for λ is 0.8, and in Ref. [3] the peak value is some 0.7. A value of 5 for h/R_a corresponds to the maximum value of 0.7 for λ seen in Ref. [6] for the aggregate crack, and is possibly a reasonable upper bound for use in l-b-b calculations.[b]

In the absence of information on h/R_a, it is suggested that in practice λ will not exceed unity.[c]

Laminar flow

For laminar flow fluid friction is given by:

$$\lambda = \frac{K}{Re} \tag{11}$$

In this instance the relationship is based on a theoretical model which also defines a value for K of 96 for flow through parallel channels. Experiments have shown that this relationship works, although values for K in excess of 96 have been seen. References [2], [3] and [6] show variations in K as a function of roughness. In each case $K = 96$ is a lower bound. In Ref. [3] a correlation with crack separation is deduced, i.e. $K = f(w)$. This relationship was based on a number of different crack surfaces. The other references also show λ to be a function of separation, and in the case of Ref. [2] roughness also. It is not possible to analyze this data to give general formulations, but it is possible to bound friction factor. The lower bound is given by $K = 96$. In Refs [2], [3] and [6] the upper bounds approximate to 600, 500 and 250, respectively. The largest of these numbers correlate very closely with an assessment based on flow through porous media.

Hence to predict laminar flow through cracks is not simple, but bounds to λ can be defined. The use of $K = 96$ will give an upper bound to flow rate. However, the calculation of the lower bound will depend on the choice of K. The program DAFTCAT uses a value for K dependant upon the crack face separation, with a maximum value of 250 or thereabouts. This aspect will be returned to in the Discussion section of this paper.

It should be noted that there is no natural upper bound to λ, and as Re goes to zero (zero flow) so λ will become infinite. Values as high as 40 for λ have been measured in experiments on flows through cracks. However, these have been associated with very low flow rates such that in conventional applications l-b-b would not have been a practical option.

Recent work at Sheffield University[8] has used CFD to investigate flow through cracks. This is ground breaking in identifying the mechanisms that effectively explain the variations seen in K in experiments. It identifies various flow regimes depending upon the proximity of roughness features as opposing surfaces approach. Of particular significance is the identification of a transition associated with the overlapping of opposing asperities. This finding is similar to that seen in the experiments reported in Ref. [6] for turbulent flow. This type of work could be extended to rationalize the differences seen in the various experiments and to formulate firm advice for various crack geometries in laminar flow.

IMPLICATION OF CALCULATED FRICTION COEFFICIENT ON FLOW RATE

The initial part of this paper drew attention to the practical difficulties in determining the actual geometry of any crack that might arise in a pressurized system. There would appear therefore to be little point in determining fluid friction to a high degree of accuracy. However, the purpose of this section is to discuss the potential consequences of uncertainty in fluid friction on a leak before break safety case.

Laminar flow

Section 3.3.2 reviewing laminar flow showed that calculated flow rate was dependent upon a constant K, and this constant varied upwards from a theoretical lower bound of 96. The maximum value determined experimentally was 600, and in this instance approached the value that might be expected in porous media.

Figure 4 shows the predicted variation in laminar flow with K. Flow rate has been normalized such that it

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[b] It is noted that Ref. [6] quotes a maximum value for λ of 4. This is for the 'Giants Causeway' surface geometry. However, as stated earlier this surface is not in anyway representative of a real crack surface.

[c] Alternatively, if f is used for fluid friction, f will not exceed 0.25.

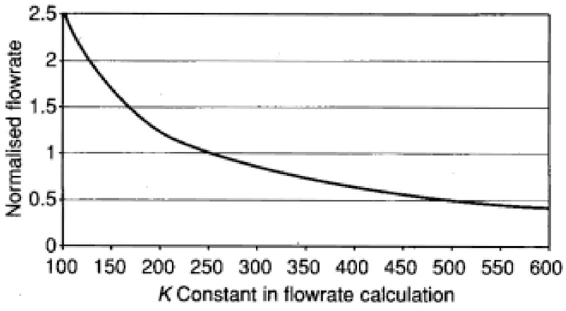

Fig. 4 Variation in laminar flow rate as a function of K.

is unity at $K=250$. This is the maximum value that arose in the experiments reported in Ref. [3] and is used here simply for illustrative purposes. It is seen that over the range of K-values considered the calculated flow rate varies by more than a factor of 6. This is large compared to the variation that will arise as a consequence of uncertainty associated with predicted crack geometry. DAFTCAT includes an empirical relationship based on crack separation in an attempt to reduce uncertainties. The correction only extends to the maximum value of K seen in the experiments supporting that program.

It should be noted that in some applications flow rates in laminar flow might be to small to be detected with confidence. On the other hand, there are situations where it is possible to detect very small leakages.

Turbulent flow

In laminar flow, it is possible to undertake rigorous calculations, as discussed in the previous section. This is not possible in turbulent flow, and in what follows an arbitrary crack geometry has been selected. Details of any calculations will change with crack geometry, but the overall picture will be somewhat similar.

As an example, for a selected crack (100 mm long, 50 µm wide in a 50-mm thick vessel) DAFTCAT has been used to calculate flow rate as a function of λ, the friction factor, and the results are shown in Fig. 5. The flow rate has then been normalized such that the flow rate is unity at a λ-value of unity. Markers are superimposed on the curve to show the predicted flow rates associated with various R_a values embracing 3–10 µm, which covers most of the range quoted in Table 1 (taken from Wilkowski *et al.*[9] and discussed subsequently). In this instance the range in flow rate is marginally greater than two. While improvements would be possible by refining estimates of surface roughness a variation in flow rate of two is probably comparable with that which would arise from uncertainties in the prediction of crack geometry.[d] However, if it is postulated that λ could be as high as four, then a further factor of two in uncertainty arises.

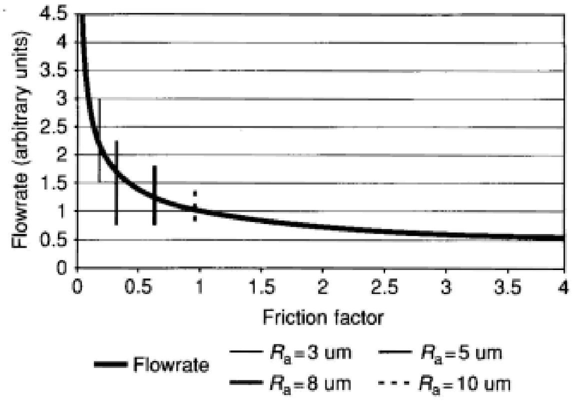

Fig. 5 Variation in flow rate as a function of friction factor.

Table 1 Summary of short wave length surface roughness values from Ref. [8]

Crack mechanism	Material	R_a range (µm)	Average R_a (µm)	Standard deviation (µm)
Inter-granular stress corrosion cracking	Stainless steel	0.64–10.5	4.7	3.9
Fatigue (air)	Stainless steel	8.1		
Fatigue (air)	Carbon steel	3–8.5	6.5	3
Corrosion fatigue	Carbon steel	3–11	8.8	3

A major contribution to the calculation of turbulent flow through cracks could come about from a firm resolution of the question related to the maximum value of λ.

Tapered cracks

In all of the data referenced previously in this paper the crack geometry has been assumed to be notionally parallel sided. In practice, this will not be the case, particularly for thick-walled pressure vessels. If the flow rate calculation code does not cope with tapered cracks then the use of mean crack separation can be used and the crack treated as though it were parallel. Figure 6 shows the results for a series of experiments through tapered cracks. A range of conditions and separations was used. Flow

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[d] Using a simple model flow is proportional to crack length, width squared, and inversely proportional to depth. An uncertainty of less than 20% in each of these parameters, and combined in the most pessimistic way, results in a factor of 2 on flow rate.

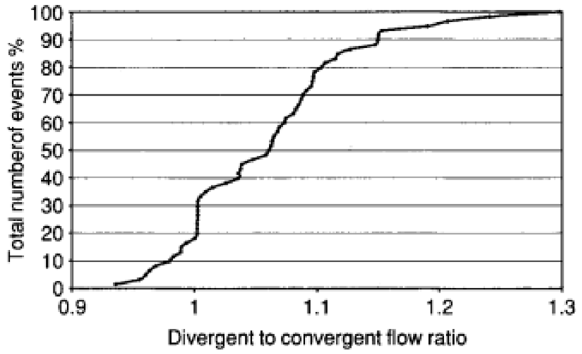

Fig. 6 Cumulative plot of flow ratio for flow through tapered cracks.

rate was measured in one direction and then the flow reversed. One flow rate was then divided by the other, and these are the data presented. It is seen that the mean difference between the two sets of data is small and the 50% point corresponds to a ratio of 1.06 with the divergent flow being the greatest. The data presented in Fig. 6 has been assessed against a simple model for flow through tapered cracks.[10] The results accord with the expected trend that flow through a divergent passage should equal, or exceed, that through a convergent one.

DISCUSSION

This paper has concentrated on determining fluid friction for the calculation of fluid flow through cracks in support of l-b-b arguments. For such purposes there are numerous uncertainties associated with the expected geometry that has to be predicted from fracture mechanics assessments. It is also important to be able to quantify the nature of the expected crack surface.

Problems associated with predicting crack geometry have been discussed. Apart from the inherent uncertainties in the prediction of the macroscopic geometry there may be the need to consider the possibility that cracks may deviate from a simple plane. This aspect can be addressed by the simple expedient of increasing the flow path length. A large number of experiments have been conducted on defective pipework and the results have identified the material and loading conditions under which cracks deviate from a principle stress axis. This information will give guidance on how to allow for increased crack length to accommodate such issues.

Reference [6] conducted experiments on rough cracks with matching surfaces and on a macroscopic scale. Measurements were made of the 'actual' surface length and compared to the nominal flow path length. The results showed the surface length to be 20% greater than nominal. Given the other uncertainties such variations in flow path length with tortuosity are small. Nevertheless, this value is probably typical of what might be expected in real cracks.

Assessments of surface roughness and tortuosity also need to be made. From the basis of the data presented in this paper it is suggested that only roughness needs to be considered. Roughness can be based on experience databases or specific measurements. There are, however, potential problems in determining R_a values for surfaces. In particular, R_a increases as the sampling length over which it is measured increases. Reference [9] identifies a range of R_a values for different crack mechanisms, and these data are reproduced in Table 1. The lower values quoted will correspond to the shortest traverse lengths and are probably the more meaningful for use in flow rate calculations. However, mean or other values can be used for sensitivity studies.

An alternative treatment is to bound the flow rate calculations. In the experimental work reported in Ref. [3] it was found that in the laminar flow regime all results did not reduce to a common line, in contrast to the predictions of classical theory. At that stage an expedient approach was taken and to bound the data a correlation was derived based on mean separation. This approach permits bounding calculations to be performed in the laminar flow regime. Recent research (Ref. [8]) using CFD has established a basis for assessing the effects of roughness on laminar flow. This is particularly the case for crack type geometries where the roughness can take up a large percentage of the separation. When roughness starts to overlap then the flow regime changes, and flow effectively tracks through the roughness rather than across the peaks.

A tool such as CFD could be of significant value in giving better estimates of flow rate and with reduced uncertainty. To model all requirements would be difficult at the moment. However, the rapid evolution of technology is expected to change this situation. Currently it would be possible to undertake parametric studies to identify the dominant features that would need to be considered in more detailed studies.

CONCLUSIONS

The calculation of flow rate through cracks in support of l-b-b arguments is subject to a number of uncertainties. The uncertainties associated with the crack surface/fluid interaction can be bounded and this paper has considered ways in which this can be done.

Empirical observations in the laminar flow regime can now be better understood given recent developments in CFD.

While uncertainty exists over modeling in turbulent flow it is suggested that a reasonable maximum value for λ is expected to be unity (or $f = 0.25$). It is possible that advances in the use of CFD to study turbulent flow through cracks could establish a firmer basis for such a postulate.

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